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Abstract

The pseudo-Goldstone boson mechanism for the "doublet-triplet splitting" problem of the grand unified theory can be naturally implemented in the scenario with extra dimensions and branes. The two SU(6) global symmetries of the Higgs sector are located on two separate branes while the SU(6) gauge symmetry is in the bulk. After including several vector-like fields in the bulk, and allowing the most general interactions with their natural strength (including the higher dimensional ones which may be generated by gravity) which are consistent with the geometry, a realistic pattern of the Standard Model fermion masses and mixings can be naturally obtained without any flavor symmetry. Neutrino masses and mixings required for the solar and atmospheric neutrino problems can also be accommodated. The geometry of extra dimensions and branes provides another way to realize the absence of certain interactions (as required in the pseudo-Goldstone boson mechanism) or the smallness of some couplings (*e.g.,* the Yukawa couplings between the fermions and the Higgs bosons), in addition to the usual symmetry arguments.

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1 Introduction

The Standard Model (SM) fermions forming complete multiplets of a single gauge group, and the unification of the $SU(3)_C$, $SU(2)_W$, and $U(1)_Y$ gauge couplings of the Minimal Supersymmetric Standard Model (MSSM) at $\sim 10^{16}$ GeV are strong suggestions that there is a grand unified gauge group (SU(5) or bigger) at a very high energy scale. However, the successful prediction of the gauge coupling unification and the proton decay constraint require the triplet partners of the two light Higgs doublets to have masses of the order of the grand unification scale. The "doublet-triplet splitting" problem is the most problematic aspect of a grand unified theory (GUT). There exist several solutions to this problem. However, to get a complete grand unified model which incorporates these solutions in a simple and appealing way is not easy.

One of the most appealing solutions to the "doublet-triplet splitting" problem is the pseudo-Goldstone bosons (PGB) mechanism [1, 2, 3], where the Higgs doublets remain light because they belong to the pseudo-Goldstone multiplets coming from breaking of the enlarged global symmetry of the Higgs superpotential. Here we briefly review it. The model is based on the gauge group SU(6). The Higgs sector consists of an adjoint (**35**), Σ, and a pair of fundamental (**6**) and anti-fundamental ($\bar{\mathbf{6}}$), H and $\bar{H}$. Provided no cross coupling exists between Σ and H, $\bar{H}$, there is an effective $SU(6)_\Sigma \times SU(6)_H$ symmetry of the Higgs sector. The Σ and H, $\bar{H}$ Higgses develop the following vacuum expectation values (vevs),

$$\langle \Sigma \rangle = \frac{1}{\sqrt{12}} \mathrm{diag}(1,1,1,1,-2,-2) v_\Sigma, \tag{1}$$

$$\langle H \rangle = \langle \bar{H} \rangle = (1,0,0,0,0,0) v_H. \tag{2}$$

These two SU(6)'s are then broken down to $SU(4)\times SU(2)\times U(1)$ and SU(5) respectively, while the SU(6) gauge symmetry is broken down to the SM gauge group $SU(3)_C \times SU(2)_\times U(1)_Y$. The successful prediction of the $\sin^2\theta_W$ is preserved if $v_H > v_\Sigma$. After counting the number of the Goldstone modes and the broken gauge generators, one finds that there are two electroweak doublets not eaten by the gauge bosons and hence left massless. They are linear combinations of the electroweak doublets coming from Σ and $H, \bar{H}$ fields,

$$h_u = \frac{v_H H_\Sigma - \sqrt{\frac{3}{4}} v_\Sigma H_H}{\sqrt{v_H^2 + \frac{3}{4} v_\Sigma^2}}, \quad h_d = \frac{v_H \bar{H}_\Sigma - \sqrt{\frac{3}{4}} v_\Sigma \bar{H}_{\bar{H}}}{\sqrt{v_H^2 + \frac{3}{4} v_\Sigma^2}}, \tag{3}$$

which parametrize the flat direction of the relative orientations of the Σ and H, $\bar{H}$ vevs. After including the soft supersymmetry (SUSY) breaking terms, the vevs will shift by an amount of the order of soft SUSY breaking parameters. This shift naturally generates a μ

term, $\mu h_u h_d$, of the same order as the soft SUSY breaking terms from the superpotential. The Higgs potential

$$V(h_u, h_d) = m_1^2 h_d^\dagger h_d + m_2^2 h_u^\dagger h_u + m_3^2 (h_u h_d + \text{h.c.}) + D\text{-terms}, \tag{4}$$

(where $m_1^2 = m_d^2 + \mu^2$, $m_2^2 = m_u^2 + \mu^2$, $m_3^2 = B\mu$, m_u^2, m_d^2 are soft SUSY breaking mass terms for the up- and down-type Higgses and B is the soft SUSY breaking parameter associated with the μ-term,) satisfies the boundary condition

$$m_1^2(M_G) = m_2^2(M_G) = -m_3^2(M_G) \tag{5}$$

at the GUT scale M_G, so that there are two massless doublet bosons corresponding to the Goldstone modes. This provide a constraint on the phenomenology of this model [2, 4]. The effective SU(6)$\times$SU(6) symmetry is explicitly broken by the couplings of matter fields to both Σ and $H\bar{H}$. The radiative corrections from these couplings lift the flat direction and it is possible to obtain the desired electroweak symmetry breaking Higgs potential after running down to the low scale [2, 4].

The problem of this model is that the cross coupling $H\Sigma\bar{H}$ is allowed by the gauge symmetry. If it exists, it destroys the SU(6)$_\Sigma\times$SU(6)$_H$ global symmetry of the Higgs sector and therefore the PGB mechanism for the light Higgs doublets. Some extra discrete symmetries or larger gauge symmetry are needed to forbid this coupling [2, 3, 5]. Besides, as one expects from the quantum gravity effects, all higher dimensional operators suppressed by the Planck scale ($M_{\rm Pl}$), which are allowed by symmetries, might be present and have $\mathcal{O}(1)$ coefficients. If that is true, then because $M_{\rm GUT}/M_{\rm Pl}$ is not a big suppression factor, the extra symmetries have to forbid the cross couplings between Σ and H, $\bar{H}$ to very high orders. This may require some unappealing symmetries or charge assignments. It would be desirable to have some better ways to suppress these unwanted couplings. As we will see in the next section, it can be naturally achieved if there are extra dimensions in which the gauge and some matter fields can propagate while Σ and H, $\bar{H}$ are localized on two separate branes. This provides a different mechanism to forbid the unwanted interactions from symmetry reasons.

Another problem of this model is how to obtain the SM fermion masses. In SU(6) models, a family of light matter fields (quarks and leptons) can be contained in $\mathbf{15} + \overline{\mathbf{6}} + \overline{\mathbf{6}}$, which is the smallest anomaly-free combination of chiral representations. However, there is no renormalizable Yukawa coupling between the light fermions belonging to $\mathbf{15} + \overline{\mathbf{6}} + \overline{\mathbf{6}}$ and the light Higgses. In order to get the large top Yukawa coupling, one can introduce a $\mathbf{20}$, a pseudo-real representation, which contains the top quark ($\mathbf{10}_3$ of SU(5)), then the top quark is naturally the only one which can receive an $\mathcal{O}(1)$ coupling from the interaction

$\mathbf{20}\,\Sigma\,\mathbf{20}$. Other fermions can get masses from the nonrenormalizable operators and therefore are naturally suppressed. However, if all nonrenormalizable operators consistent with the gauge symmetry exist, a realistic fermion mass pattern is not obtained. Therefore, one also has to introduce extra discrete symmetries and assume that the higher dimensional operators are generated by integrating out some heavy vector-like fields [6, 5] in order to obtain a realistic pattern of fermion masses and mixings. In section 3 we will find that the geometry of extra dimensions and branes for the PGB mechanism can also help to explain the fermion mass hierarchies without appealing to flavor symmetries.

2 Doublet-triplet splitting in extra dimensions

Now let us discuss how the doublet-triplet splitting and fermion mass hierarchies can naturally arise in the scenario with extra dimensions and branes. We assume that the SU(6) gauge field propagates in the bulk of a $4+n$ dimensional space-time with n dimensions of space compactified with a radius R. The two kinds of Higgses Σ and H, $\bar{H}$ on the other hand are localized on two parallel 3-branes separated by a distance r in the $4+n$ dimensions, so there is no direct interaction between them. Extra dimensions with compactification radius larger than the Planck (or string) length have been considered in string theory [7, 8, 9, 10]; they have been used to lower the unification scale [11]; a very large compactification radius can even push the fundamental Planck scale, M_*, down to $\mathcal{O}(\text{TeV})$, providing an alternative solution to the hierarchy problem [12, 13, 14]. In this paper we consider the compactification of extra dimensions occurs at high energies, around the GUT scale, so that the successful gauge coupling unification still works in the traditional way.[1] We assume that the distance between these two 3-branes, r, is much smaller than the compactification radius R, but larger than the fundamental Planck distance $1/M_*$, so that we can still use the field theory description

[1]In fact, the simple SUSY GUT prediction of the strong coupling constant is a little higher than the experimental value. If $1/R$ is smaller than $M_{\rm GUT}$, the contribution from the Kaluza-Klein states of the gauge fields will lower the prediction of the strong coupling constant, so it may be favorable to have $1/R$ a little bit lower than $M_{\rm GUT}$. We will not discuss this in details. See the Refs. [11, 15] for the discussions of gauge coupling unification.

without dealing with the full quantum gravitational theory.[2] Therefore we have

$$M_{\rm GUT} \lesssim \frac{1}{R} < \frac{1}{r} < M_* < M_{\rm Pl}, \tag{6}$$

where $M_{\rm Pl} \simeq 2.8 \times 10^{18}$GeV is the effective 4-dimensional Planck scale. If there are no additional large extra dimensions in which gravitons propagate, $M_{\rm Pl}$ is related to M_* and R by [12, 13]

$$\frac{M_{\rm Pl}^2}{M_*^2} = M_*^n R^n. \tag{7}$$

This relation can be modified if there are additional large dimensions in which gravitons propagate.

On these two branes, we assume for simplicity that the Higgs superpotential takes the simple form,

$$\begin{aligned} W_1 &= m_\Sigma \Sigma^2 + \Sigma^3, && (8) \\ W_2 &= S(H\bar{H} - v_H^2), && (9) \end{aligned}$$

where S is a singlet field, so that Σ and H, $\bar{H}$ acquire vevs of the form given by Eqs.(1),(2). To preserve the gauge coupling unification we need

$$M_{\rm GUT} = v_\Sigma < v_H \, (< M_*), \tag{10}$$

so that the light Higgses are predominantly contained in Σ. How exactly they acquire such vevs is not important, and they may be generated dynamically [16].

Most of the SM matter fields as well as some additional heavy vector-like fields live in the bulk. We assume that the extra dimensions are compactified on an orbifold so that the unwanted zero modes are projected out and we can get chiral multiplets in four dimensions. The $\mathrm{SU}(6)_\Sigma \times \mathrm{SU}(6)_H$ global symmetry on these two branes is broken by the couplings of the matter fields living in the bulk to the Higgses on both branes. Nevertheless, if we assign a matter parity (which is equivalent to the R-parity of the MSSM) -1 to all fields living in the bulk, and $+1$ to the Higgses Σ and H, $\bar{H}$, then no direct superpotential couplings between Σ and H, $\bar{H}$ (and containing no matter fields) at any order can be generated after integrating out the extra dimensions. Thus, the PGB mechanism for the doublet-triplet splitting can work naturally in this scenario.

[2]The validity of the field theory description may be questioned at the scale very close to M_*. However, without knowing how to describe the full quantum gravitational theory, we assume that just beneath M_*, physics can be described by the usual field theory with the gravity effects included in the higher dimensional interactions suppressed by M_*.

3 Fermion masses

Before getting into the details of the fermion masses and mixings in the Standard Model, we first discuss in general the possible suppressions of couplings we may get in such a scenario. In addition to the usual Planck mass suppression for the higher dimensional operators, the suppressions may also come from the large volume factor of the extra dimensions and from integrating out the vector-like bulk fields and their Kaluza-Klein excitations.

The couplings of an (external) bulk field (after integrating out the extra dimensions and heavy fields) to the brane fields are suppressed by the volume factor of the extra dimensions. The zero modes contain an $R^{-n/2}$ factor after the Fourier decomposition to match the mass dimensions of fields in different space dimensions, so the dimensionless coefficients of the couplings are naturally suppressed by [13, 17]

$$\epsilon \equiv (M_* R)^{-\frac{n}{2}} \left(= \frac{M_*}{M_{\rm Pl}}, \text{ if no additinal large dimensions for gravitons}\right). \tag{11}$$

This may explain the weakness of the unified gauge coupling at the GUT scale. To get $\mathcal{O}(1)$ Yukawa coupling for the top quark, we therefore assume that the **20** (denoted by η, with matter parity -1) containing the top quark lives on the same brane in which Σ resides, then the $\eta\,\Sigma\,\eta$ interaction which contains the top Yukawa coupling is naturally $\mathcal{O}(1)$. All other matter and vector-like fields are assumed to live in the bulk.

Light fermion masses come from higher dimensional operators. Higher dimensional operators can already be present in the fundamental Lagrangian (suppressed by powers of M_*) if they involve fields in the bulk and on one brane only. They can also be generated by integrating out the heavy vector-like fields in the bulk and extra dimensions if they contain fields on both branes. This is somewhat similar to the Froggatt-Nielsen mechanism [18]. However, the suppression of these higher dimensional operators is different. It also depends on the transverse distance r between the two branes and the number of extra dimensions, as there is a tower of the Kaluza-Klein states of the vector-like fields. The case when there are vector-like scalars connecting two branes is discussed in Ref. [17]. It is simply the Yukawa potential (or the propagator of the vector-like field) in the n transverse direction. The generalization to the supersymmetric case is straightforward. The propagator in the transverse direction is

$$\Delta_V(r) = \int d^n\kappa\, e^{i\kappa r} \frac{-i\kappa + m_V}{\kappa^2 + m_V^2}. \tag{12}$$

One gets an exponential suppression ($e^{-m_V r}$) if the mass of the vector-like fields m_V is larger than $1/r$, and a power suppression (r^{1-n}) if m_V is smaller than $1/r$. For one extra dimension and $m_V < 1/r$, there can even be no suppression. We will parametrize the dimensionless suppression factor (in the unit of M_*) by δ_V (*e.g.*, $(M_* r)^{-a}$ in the case of power suppression). We will find that to obtain successful fermion masses some of the suppression factor from integrating out the vector-like fields should be $\mathcal{O}(1)$, so it is favorable to have just one extra dimension.

In the bulk, there are three sets of $\mathbf{15} + \overline{\mathbf{6}} + \overline{\mathbf{6}}$ chiral matter multiplets, denoted by $\psi_i(\mathbf{15})$, $\bar{\phi}_i(\bar{\mathbf{6}})$, $\bar{\phi}'_i(\bar{\mathbf{6}})$, $i = 1, 2, 3$. In addition, we assume that there are 3 pairs of vector-like fields of the SU(6) representations $(\mathbf{20}_1, \mathbf{20}_2)$, $(\mathbf{6}, \overline{\mathbf{6}})$, and $(\mathbf{70}, \overline{\mathbf{70}})$ with masses (of the zero modes) m_{20}, m_6, and m_{70} respectively. They all have matter parity -1. The field content is summarized in Table 1. In terms of the usual $\mathrm{SU}(5)_{\mathrm{GUT}}$ subgroup, these representations

Brane 1	Bulk	Brane 2
Σ	SU(6) gauge field, ψ_i, $\bar{\phi}_i$, $\bar{\phi}'_i$, $i = 1, 2, 3$	H, $\bar{H}$
η	$\mathbf{20}_1, \mathbf{20}_2$, $\mathbf{6}, \overline{\mathbf{6}}$, $\mathbf{70}, \overline{\mathbf{70}}$	S, (N)

Table 1: Field content in the bulk and on the two branes. As it will be discussed later, the singlet field N is included if we need to generate the neutrino mass to account for the atmospheric neutrino oscillation.

decompose into:

$$\begin{aligned}
\mathbf{6} &= \mathbf{1} + \mathbf{5}, \quad \overline{\mathbf{6}} = \mathbf{1} + \overline{\mathbf{5}}, \\
\mathbf{15} &= \mathbf{5} + \mathbf{10}, \\
\mathbf{20} &= \mathbf{10} + \overline{\mathbf{10}}, \\
\mathbf{35} &= \mathbf{1} + \mathbf{5} + \overline{\mathbf{5}} + \mathbf{24}, \\
\mathbf{70} &= \mathbf{5} + \mathbf{10} + \mathbf{15} + \overline{\mathbf{40}}, \quad \overline{\mathbf{70}} = \overline{\mathbf{5}} + \overline{\mathbf{10}} + \overline{\mathbf{15}} + \mathbf{40},
\end{aligned} \tag{13}$$

Integrating out the vector-like fields and the extra dimensions, we obtain the operators appearing in the effective four dimensional theory. The dimensionless coefficient (after factorizing out powers of M_* of the dimensionful coupling) of an operator is suppressed by a power of ϵ for each external bulk field, and by δ_V if it is generated by integrating out the vector-like fields $V, \overline{V}$. For Yukawa couplings coming from nonrenormalizable operators, they will also be suppressed by v_H/M_* or v_Σ/M_*. If the light Higgs doublets come from H, $\bar{H}$, there is a further suppression of the mixing angle $\sim v_\Sigma/v_H$. In the following we discuss these effective operators and the SM fermion masses and mixings arising from them.

Operators which decouple the extra states (Fig. 1):

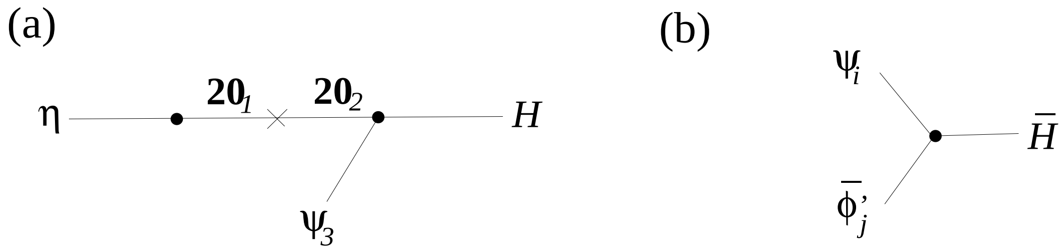

Figure 1: Diagrams which decouple the extra states. Fields on the left of the left interaction point are on brane 1. Fields on the right of the right interaction point are on brane 2. Fields between the interaction points are in the bulk.

- $\eta H \psi_3$ (diagram (a)): We can define ψ_3 with this operator by using the rotation freedom among ψ_i's, and $\mathbf{20}_1$ to be the one which couples to η. The $\mathbf{10}$ (of $\mathrm{SU}(5)_{\mathrm{GUT}}$) in ψ_3 and $\overline{\mathbf{10}}$ in η become heavy due to $\langle H \rangle = v_H$, leaving only three $\mathbf{10}$'s (in η, ψ_2, ψ_1) in the low energies.
- $\psi_i \bar{H} \bar{\phi}'_j$ (diagram (b)): The $\mathbf{5}$'s in ψ_i and $\overline{\mathbf{5}}$'s in $\bar{\phi}'_j$ are married by $\langle \bar{H} \rangle$, leaving only three $\overline{\mathbf{5}}$'s (in $\bar{\phi}_i$) in the low energies.

Because of the suppresion factors involved, some decoupled states will have masses a little bit lower than M_{GUT}. However, they are complete SU(5) multiplets, so they do not affect the coupling unification. We can see that ψ_3 is completely decoupled, so we will drop it in the following discussion. In SU(5) notation, the three light generations are contained in the $\mathbf{10}$'s of η, ψ_2, ψ_1, and $\overline{\mathbf{5}}$'s of $\bar{\phi}_3$, $\bar{\phi}_2$, $\bar{\phi}_1$. They are the only SM non-singlets matter fields left massless at this stage. (The SU(5) singlets can also be decoupled. It will be seen when we discuss neutrino masses.)

Up-type quark masses (Fig. 2):

- $\eta \Sigma \eta$ (diagram (c)): It contains $\mathbf{10}_3 \mathbf{5}_\Sigma \mathbf{10}_3$ in SU(5) notation. There is no suppression and therefore it gives an $\mathcal{O}(1)$ Yukawa coupling to the top quark.
- $\eta \Sigma \psi_2 H$ (diagram (d)): One can rotate ψ_i, $i = 1, 2$, so that only ψ_2 couples to $\overline{\mathbf{70}}$ and H. It generates the 23 and 32 elements of the up Yukawa matrix of $\mathcal{O}(\epsilon \delta_{70} (\frac{v_H}{M_*}))$.
- $\eta \psi_i H (H \bar{H})$ (diagram (e)): It does not contain Σ, so the light Higgs has to come from H, which causes a $(\frac{v_\Sigma}{v_H})$ mixing suppression. It generates 13, 31, 23, 32 elements of $\mathcal{O}(\epsilon \delta_{20} (\frac{v_H}{M_*})^2 (\frac{v_\Sigma}{v_H}))$.

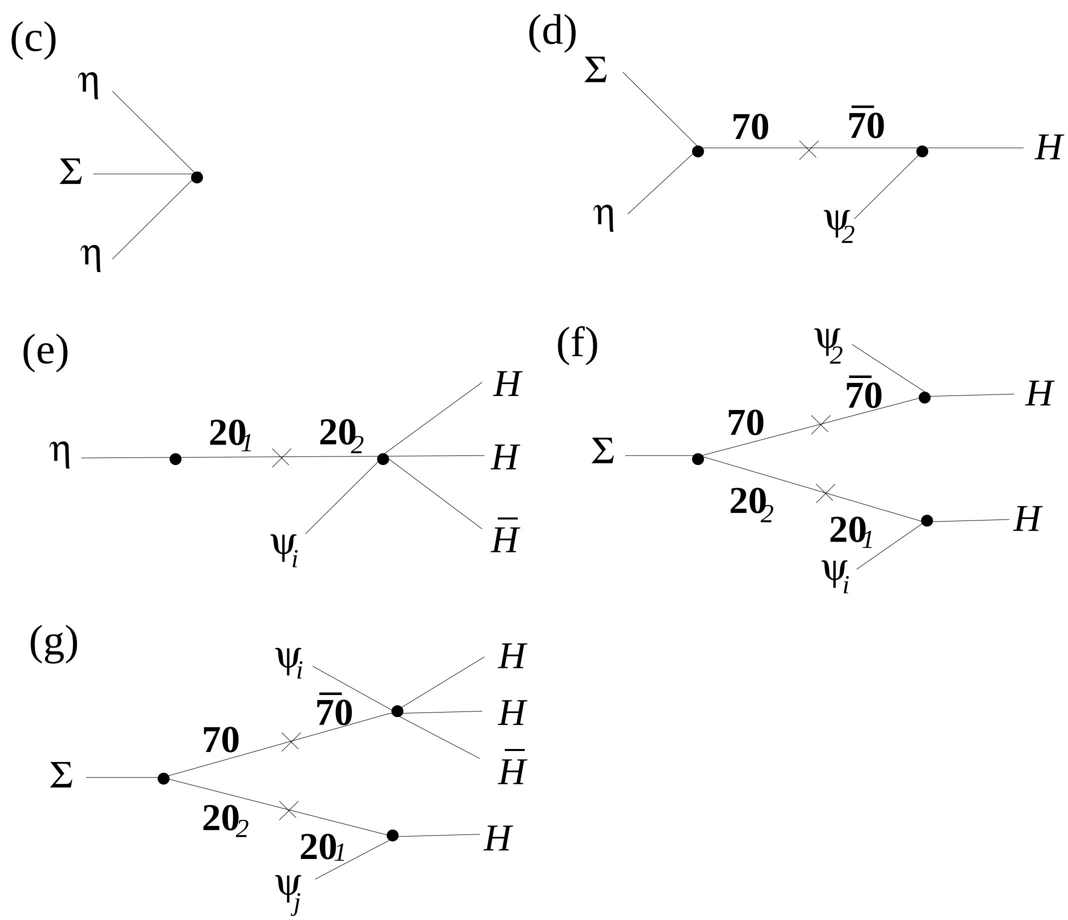

Figure 2: Diagrams which generate the up-type quark masses.

- $\Sigma\psi_2\psi_i HH$ (diagram (f)): It generates 22, 12, 21 elements of $\mathcal{O}(\epsilon^2\delta_{20}\delta_{70}(\frac{v_H}{M_*})^2)$.
- $\Sigma\psi_i\psi_j HH(H\bar{H})$ (diagram (g)): One can always attach a pair of $(H\bar{H})$ to the interaction on the brane 2, which will be suppressed by an extra $(v_H/M_*)^2$. This gives the leading contribution to the 11 element.

In the leading order the up-type Yukawa matrix looks like

$$\lambda_U \sim \begin{pmatrix} u_4 & u_3' & u_2 \\ u_3' & u_3 & u_1 \\ u_2 & u_1 & 1 \end{pmatrix}, \tag{14}$$

where

$$u_1 \sim \epsilon\delta_{70}\left(\frac{v_H}{M_*}\right), \tag{15}$$

$$u_2 \sim \epsilon\delta_{20}\left(\frac{v_H}{M_*}\right)^2\left(\frac{v_\Sigma}{v_H}\right), \tag{16}$$

$$u_3,\, u_3' \sim \epsilon^2\delta_{20}\delta_{70}\left(\frac{v_H}{M_*}\right)^2, \tag{17}$$

$$u_4 \sim \epsilon^2\delta_{20}\delta_{70}\left(\frac{v_H}{M_*}\right)^4. \tag{18}$$

If we take $\epsilon \sim \frac{1}{3}$, $\frac{v_H}{M_*} \sim \frac{1}{5}$, $\frac{v_\Sigma}{v_H} \sim \frac{1}{3}$, $\delta_{70} \sim \frac{1}{2}$, and $\delta_{20} \sim \frac{1}{40}$, then we have at the GUT scale,

$$\lambda_t \sim 1, \tag{19}$$

$$\lambda_c \sim u_1^2 \sim \epsilon^2\delta_{70}^2\left(\frac{v_H}{M_*}\right)^2 \sim 10^{-3}, \tag{20}$$

$$\lambda_u \sim u_4,\, \frac{u_3'^2}{u_1^2}\,\text{(two comparable contributions)} \sim (2-3)\times 10^{-6}. \tag{21}$$

Remember that the light fermion Yukawa couplings will increase in renormalization group (RG) running to low energies while the top Yukawa coupling will roughly approach some fixed point. The mass ratios of light quarks to the top quark will be enhanced by a factor of 5–10 relative to those at the GUT scale. After taking into account the RG effect, the above numbers give a good approximation to the up-type quark masses. In diagonalizing the mass matrix, the 23 rotation angle $U_{23} \sim u_1 \sim 3\times 10^{-2}$ is about the same order as V_{cb}. Other rotation angles are much smaller than the corresponding Cabibbio-Kobayashi-Maskawa (CKM) matrix elements, so they have to be generated from the down sector.

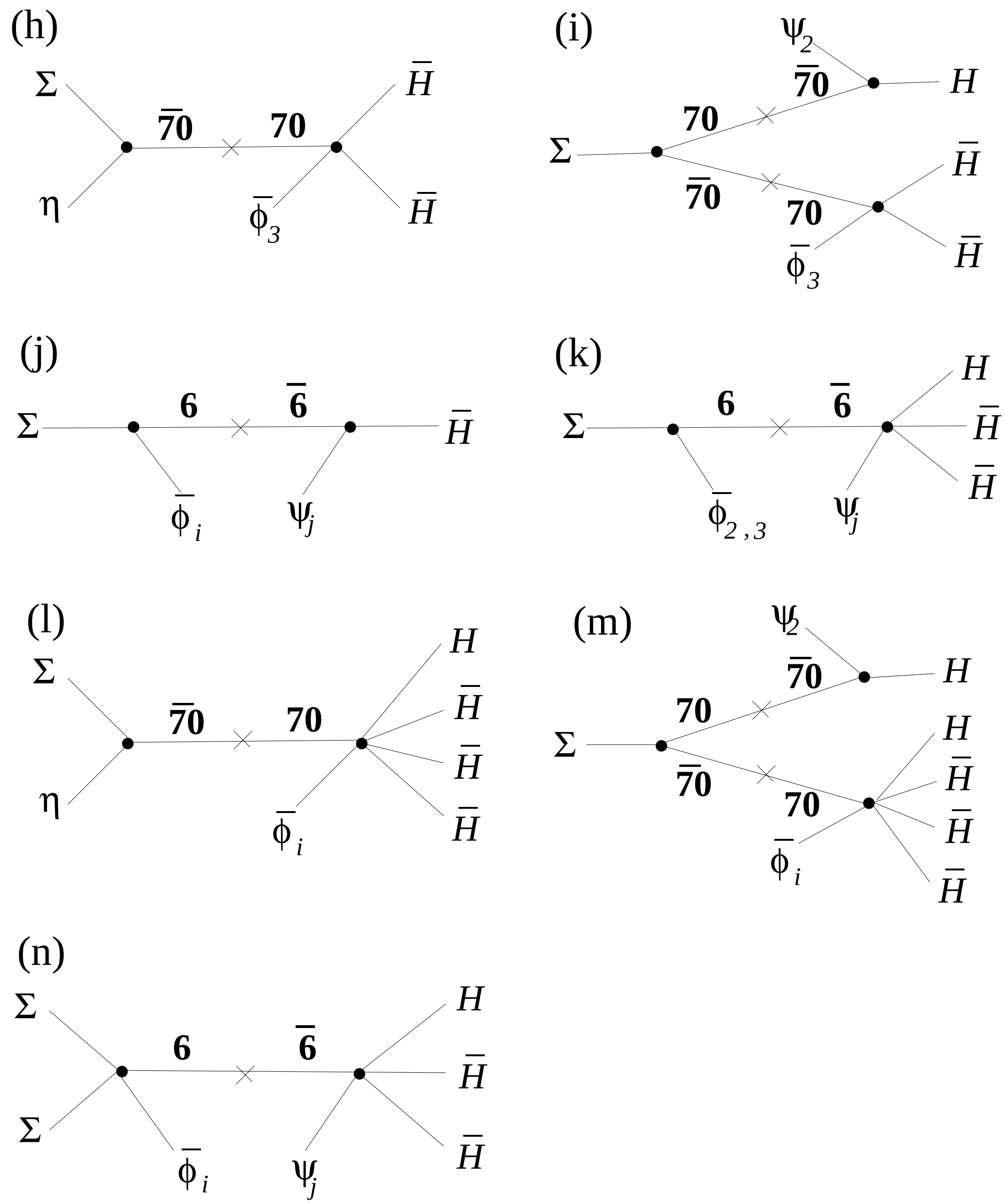

Figure 3: Diagrams which generate the down-type quark and charged lepton masses.

Down-type quark and charged lepton masses (Fig. 3):

- $\eta\Sigma\bar{\phi}_3\bar{H}\bar{H}$ (diagram (h)): We can use the rotation freedom among $\bar{\phi}_i$'s to define $\bar{\phi}_3$ with this operator. This gives the 33 elements of the down and charged lepton Yukawa matrix. In leading order, the Higgs doublet to which the fermions couple comes from Σ. They are $\mathcal{O}(\epsilon\delta_{70}(\frac{v_H}{M_*})^2)$ and the same for the down-type quark and the charged lepton, so we have appproximate $b-\tau$ unification.
- $\Sigma\psi_2 H\bar{\phi}_3\bar{H}\bar{H}$ (diagram (i)): ψ_2 and $\bar{\phi}_3$ have been defined before. This operator contributes to the 23 elements of the down and lepton Yukawa matrices and is $\mathcal{O}(\epsilon^2\delta_{70}^2(\frac{v_H}{M_*})^3)$.
- $\langle\Sigma\rangle\bar{\phi}_i\psi_j\bar{H}$ (diagram (j)): This operator only redefines $\bar{\phi}'_i$ and is irrelevant for fermion masses [6].
- $\langle\Sigma\rangle\bar{\phi}_i\psi_j\bar{H}(H\bar{H})$ (diagram (k)): Attaching $(H\bar{H})$ to the previous diagram, we can get a diagram contributing to the fermion masses. We can rotate $\bar{\phi}_i$ to have only $\bar{\phi}_2$, $\bar{\phi}_3$ in this operator. If we had not defined ψ_i's in the up sector, we could also have defined ψ_2 by this operator, then it would have contributed only to the 22 and 23 elements of the mass matrices. The rotation angle between the two bases will in general be $\mathcal{O}(1)$, which accounts for why the Cabibbo angle is large. In the basis used for the up sector, it contributes to the 12, 13, 22, 23 elements of the down and charged lepton mass matrices (with 12, 13 elements smaller than 22, 23 elements by $\sim \sin\theta_C \sim 0.2$). An important fact of this operator is that the intermediate states ($\mathbf{6}$, $\overline{\mathbf{6}}$) do not contain $\mathbf{10}$, $\overline{\mathbf{10}}$ of $\mathrm{SU}(5)_{\mathrm{GUT}}$, so the light Higgs doublet has to come from $\bar{H}$. The contribution of this operator to the Yukawa couplings is therefore $\mathcal{O}(\epsilon^2\delta_6(\frac{v_\Sigma}{M_*})(\frac{v_H}{M_*})^2(\frac{v_\Sigma}{v_H}))$. The vev of Σ gives a ratio of $1:-2$ to the down-type quark and the charged lepton Yukawa matrix elements. Since it is the dominant term to the 22 elements and hence the leading contribution to the second generation masses, this offers an explanation of the discrepancy between m_s and m_μ from the simple unification relation.

Other matrix elements and non-leading contributions can be obtained by attaching more $(H\bar{H})$ or Σ to previous diagrams. In the following we only discuss the leading contributions.

- $\eta\Sigma\bar{\phi}_i\bar{H}\bar{H}(H\bar{H})$ (diagram (l)): This gives the leading contribution to the 31, 32 elements of the down and lepton mass matrices of $\mathcal{O}(\epsilon\delta_{70}(\frac{v_H}{M_*})^4)$.

- $\Sigma\psi_2 H\bar{\phi}_i\bar{H}\bar{H}(H\bar{H})$ (diagram (m)): This gives the leading contribution to the 21 elements of the down and lepton mass matrices of $\mathcal{O}(\epsilon^2\delta_{70}^2(\frac{v_H}{M_*})^5)$.
- $\langle\Sigma\rangle^2\bar{\phi}_i\psi_j\bar{H}(H\bar{H})$ (diagram (n)): This gives the leading contribution to the 11 elements of $\mathcal{O}(\epsilon^2\delta_6(\frac{v_\Sigma}{M_*})^2(\frac{v_H}{M_*})^2(\frac{v_\Sigma}{v_H}))$.

In the leading order the down-type Yukawa matrix looks like

$$\lambda_D \sim \begin{pmatrix} d_6 & sd_3 & sd_3' \\ d_5 & d_3 & d_2(+d_3') \\ d_4 & d_4 & d_1 \end{pmatrix}, \tag{22}$$

where

$$d_1 \sim \epsilon\delta_{70}\left(\frac{v_H}{M_*}\right)^2, \tag{23}$$
$$d_2 \sim \epsilon^2\delta_{70}^2\left(\frac{v_H}{M_*}\right)^3, \tag{24}$$
$$d_3,\, d_3' \sim \epsilon^2\delta_6\left(\frac{v_\Sigma}{v_H}\right)^2\left(\frac{v_H}{M_*}\right)^3, \tag{25}$$
$$d_4 \sim \epsilon\delta_{70}\left(\frac{v_H}{M_*}\right)^4, \tag{26}$$
$$d_5 \sim \epsilon^2\delta_{70}^2\left(\frac{v_H}{M_*}\right)^5, \tag{27}$$
$$d_6 \sim \epsilon^2\delta_6\left(\frac{v_\Sigma}{v_H}\right)^3\left(\frac{v_H}{M_*}\right)^4, \tag{28}$$

and we have explicitly put in the Cabibbo angle $s \sim 0.2$. Again, taking the previous assumed suppression factors, $\epsilon \sim \frac{1}{3}$, $\frac{v_H}{M_*} \sim \frac{1}{5}$, $\frac{v_\Sigma}{v_H} \sim \frac{1}{3}$, $\delta_{70} \sim \frac{1}{2}$, $\delta_{20} \sim \frac{1}{40}$, with $\delta_6 \sim 1$, we have the following relations at the GUT scale,

$$\frac{\lambda_b}{\lambda_t} \sim \epsilon\delta_{70}\left(\frac{v_H}{M_*}\right)^2 \sim 10^{-2}, \tag{29}$$
$$\frac{\lambda_s}{\lambda_b} \sim \frac{d_3}{d_1} \sim \epsilon\frac{\delta_6}{\delta_{70}}\left(\frac{v_\Sigma}{v_H}\right)^2\left(\frac{v_H}{M_*}\right) \sim 1.5\times 10^{-2}. \tag{30}$$

In running down to low energies, we get enhancements of $\sim 3-5$ for $\frac{\lambda_b(m_b)}{\lambda_t(m_t)}$ and ~ 2 for $\frac{\lambda_s(1\mathrm{GeV})}{\lambda_b(m_b)}$. There are several comparable leading contributions to λ_d after diagonalization, *e.g.*, d_6, $(sd_3d_2d_4)/(d_1d_3)$, $(sd_3d_1d_5)/(d_1d_3)$. In terms of $\frac{\lambda_d}{\lambda_s}$, they are

$$\frac{d_6}{d_3} \sim \left(\frac{v_\Sigma}{v_H}\right)\left(\frac{v_H}{M_*}\right) \sim \frac{1}{15}, \tag{31}$$

$$\frac{sd_3 d_2 d_4}{d_1 d_3^2} \sim \frac{s\delta_{70}^2 \left(\frac{v_H}{M_*}\right)^2}{\delta_6 \left(\frac{v_\Sigma}{v_H}\right)^2} \sim 2 \times 10^{-2}, \tag{32}$$

$$\frac{sd_3 d_1 d_5}{d_1 d_3^2} \sim \frac{s\delta_{70}^2 \left(\frac{v_H}{M_*}\right)^2}{\delta_6 \left(\frac{v_\Sigma}{v_H}\right)^2} \sim 2 \times 10^{-2}. \tag{33}$$

The smallness of λ_e may due to a mild cancellation among these contributions. We can see that we also get a very good approximation of the down quark and charged lepton masses (for small $\tan\beta$).

The rotation angles for diagonalizing the down quark mass matrix are

$$D_{23} \sim \frac{d_2}{d_1} \sim \epsilon\delta_{70}\left(\frac{v_H}{M_*}\right) \sim 3 \times 10^{-2}, \tag{34}$$

$$D_{13} \sim \frac{sd_3'}{d_1} \sim s\epsilon\frac{\delta_6}{\delta_{70}}\left(\frac{v_\Sigma}{v_H}\right)^2\left(\frac{v_H}{M_*}\right) \sim 3 \times 10^{-3}, \tag{35}$$

$$D_{12} \sim s \sim 0.2\ (\mathcal{O}(1)). \tag{36}$$

U_{23} and D_{23} are comparable and their combination gives V_{cb}. Other 'CKM matrix elements are dominated by the rotation of the down sector and they are all generated at the right magnitudes. It is quite remarkable that without any extra flavor symmetry and allowing most general operators, the SM fermion masses and mixings pattern is naturally obtained provided the various mass scales are such that they produce the appropriate suppression factors.

Neutrino masses (Fig. 4):

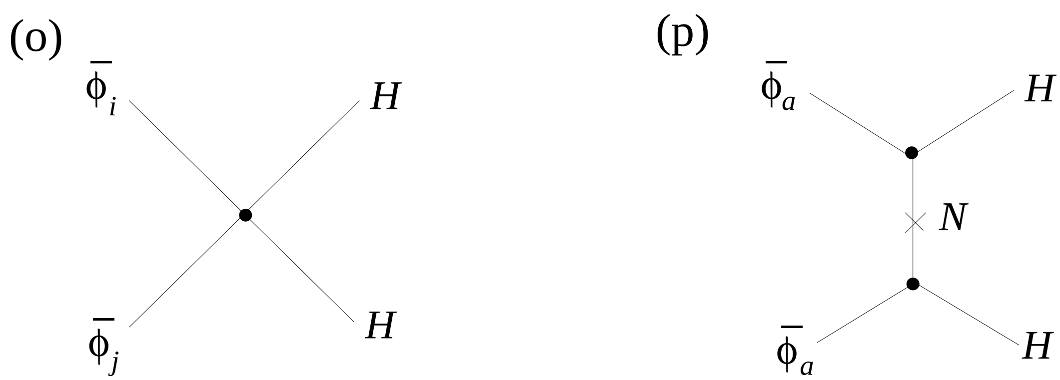

Figure 4: Diagrams which generate the neutrino masses.

Majorana masses of the left handed neutrinos can be generated by diagram (o). This diagram also decouples the SU(5) singlets in $\bar\phi_i$ (and also in $\bar\phi_i'$ by replacing $\bar\phi_i$ by $\bar\phi_i'$).

Because there is no distinction among the three generations of $\bar{\phi}_i$, in general we have large mixings among the neutrinos. The neutrino masses generated by this diagram are of the order $\sim \epsilon^2 (\frac{v_\Sigma}{v_H})^2 \frac{v_{\rm EW}^2}{M_*} \sim 10^{-5} - 10^{-6}$ eV. This is in the right range of explaining the solar neutrino problem through the "just-so" vacuum oscillation solution [19], but too small to account for the atmospheric neutrino problem, which requires $\delta m_{\rm atm} \sim 3 \times 10^{-2} - 10^{-1}$ [20, 21]. To accommodate a larger neutrino mass, one can introduce a singlet field N (with matter parity -1) on brane 2 (which contains $H\bar{H}$). Then, from diagram (p), one neutrino mass of the order $\sim \epsilon^2 (\frac{v_\Sigma}{v_H})^2 \frac{v_{\rm EW}^2}{M_N}$ can be generated. It will be in the right range for the atmospheric neutrino problem if the mass of the singlet mass M_N is $\sim 10^{13} - 10^{14}$ GeV. The next neutrino mass obtained from attaching $(H\bar{H})$'s to diagram (p) will be suppressed by $(\frac{v_H}{M_*})^4 \sim 10^{-3}$, close to that required for the vacuum oscillation solution of the solar neutrino problem.

4 Conclusions

In conclusion, extra dimensions and fields localized on branes provide a new way to understand the absence or smallness of some couplings without symmetry arguments [17, 22, 23, 24]. This kind of idea has been used to obtain small fermion masses in the Standard Model and to suppress proton decay [24]. Here we find that by localizing two kinds of Higgses on two separate branes, the most difficult "doublet-triplet splitting" problem of the grand unified theory is naturally solved by the pseudo-Goldstone boson mechanism. In addition, after including several vector-like fields in the bulk, and allowing the most general interactions consistent with the background geometry and with their natural strength, all Standard Model fermion masses and mixings can be correctly produced without any flavor symmetry. The neutrino masses and mixings required for the solar and atmospheric neutrino problems can also be easily accommodated. It is very interesting that the complicated picture of the Standard Model can be realized by such a simple model. Extra dimensions at such high energies will not give us the exciting new collider signatures such as production of the graviton Kaluza-Klein states. Nevertheless, it gives a simple realization of the grand unified theory and the fermion masses with the pseudo-Goldstone boson solution to the "doublet-triplet splitting" problem. If it is true, the boundary condition of the Higgs parameters should be verified in the future experiments.

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